

The Liar-paradox in a Quantum Mechanical Perspective*

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Abstract

In this paper we concentrate on the nature of the liar paradox as a cognitive entity; a consistently testable configuration of properties. We elaborate further on a quantum mechanical model [Aerts, Broekaert, Smets 1999] that has been proposed to analyze the dynamics involved, and we focus on the interpretation and concomitant philosophical picture. Some conclusions we draw from our model favor an effective realistic interpretation of cognitive reality.

1 Introduction.

Our approach, to analyze the Liar paradox as a cognitive entity, emerges naturally from research on integrating world views (Worldviews Group, 1994, 1995; Broekaert, 1999). The justification is based on their necessary inclusion in an encompassing model. An integrating world view can be based on a model of interacting layers epistemically corresponding to the various contemporary sciences (CLEA, 1997). Basically, in the same way as we view social entities in a social layer, or quantum entities in the quantum — which in this context can be viewed as the pre-material — layer, we place cognitive entities in a cognitive layer of reality. Which is considered the expanse where the personal and interpersonal cognitive interactions are taking place, and which will be elaborated in the next section.

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In this paper, we will work with versions of the Liar Sentence with index or sentence pointers followed by the sentence this index points at. Typically we will either concentrate on the single sentence version;

‘Single Liar’

(1) sentence (1) is false

or the two sentence version;

‘Double Liar’

(1) sentence (2) is false

(2) sentence (1) is true

First, we stress that it is not our intention to construct a solution for the paradox, but we will concentrate in this paper on a model of the self-referential circularity — more precisely, the truth-value dynamics — involved. In this way, we are able to understand the nature of the liars’ paradoxicality by modelling the typical truth-value oscillation in the encompassing framework. By taking this position there is no need to go into details of ‘solving’-strategies that have been proposed in the literature. Actually, the most important of these strategies are specifically constructed to avoid the paradox by, for example, declaring it ‘ungrammatical’, or ‘meaningless’ or ‘ungrounded’. All these strategies, of which we only mentioned some, and those that introduced truth value ‘gaps’ or ‘gluts’, have proven valuable, and serve here to distinguish the present position. For more details on the mentioned solving strategies, see for example (Grim 1991, chp 1).

Finally, we mention some issues concerning the recourse taken to the quantummechanical formalism in describing cognitive entities. Contrary to the present paper, many attempts have been made to approach ‘emergent’ phenomena — like cognition — through complex dynamics. Focusing on the collective dynamics of quite different entities, some very similar patterns are encountered regardless of the nature of the substrate. This has been pointed out in research pertaining to the theories of nonlinearity and complexity. Because of the efficacy of these theories, tentative conclusions about the nature of — quite disparate — processes in reality can be inferred from structural-dynamical similarities. Although, we have to stress that these similarities generally involve only so called ‘noncontextual’ entities, as the theories of nonlinearity and complexity are fundamentally classical theories. By noncontextuality we mean that the entity will ‘not’ be influenced by an

act of observation (measurement of the entity). A direct consequence of the fact that the theories of nonlinearity and complexity are classical deterministic. It is now our claim that cognitive entities — especially of the type of the liar paradox — share with quantum entities a variable, and sometimes high, contextual nature: i.e. also cognitive entities, as it is the case for quantum entities, are influenced by the act of measurement, which in the case of a cognitive entity is generated by the cognitive interaction. Therefore the theory of quantum mechanics can be used in analyzing the nature of cognitive entities and more specifically the nature of the liar paradox. In (Aerts, Broekaert, Smets, 1999) we built a quantum mechanical model that fits this purpose. With respect to the high contextual nature of intricate cognitive and also social entities, we can point out an approach where the probability model that results in an opinion pole — with important influence of the interviewer on the interviewee — is of a quantum mechanical nature (Aerts, 1998; Aerts and Aerts, 1995, 1996; Aerts, Coecke and Smets, 1999). In a similar way, we encounter an important contextual influence of an observer reading (observing) the liar paradox sentences. In this case the high contextuality will occur through the observer — the cognitive person — reasoning through the self-referent entity.

2 Cognitive Entities

The existence of a cognitive entity is recognized by its aptitude of being generally and cognitively influenced on as a practically stable configuration, e.g. in the reasoning on it and communicating about it, and by the limited number of different states that it can be in. The cognitive entity is endowed with properties and relations with the other elements of its layer. Their internal coherence relates to language as well as conceptions of experience. The extent to which it is related to pendants in the physical and other layers enhances its identity and its granted coincidence with the physically real entity. This variable correspondence relation is put forward between entities of the ontological cognitive layer and their pendants in physical or other layers.

In order to give a more precise and complete characterization of the cognitive entity, we need to reconstruct its interaction profile. This is done by specifying the different kinds of measurements or experiments appropriate to the intended entity. This necessarily happens with the intervention of the cognitive person — the conscious human being conceiving the cognitive entity.

We distinguish between those entities that refer to elements of layers other than the cognitive one and entities that do not. Arguably, such a distinction may only be possible in specific phases of cognition and theory forming, respecting personal psychological evolution. In both cases, the influencing or intervening of the cognitive person will be different; engendering physical interventions or only cognition, reasoning.

We define such an intervention as a kind of measurement on the entity with the aim of gaining information or increasing knowledge on its precise state. In order to keep our analogy with the physical layer, we restrict our exposition here to measurements that characterize the state of a cognitive entity.

The state formalizes specific — or all — possible actualisations according the interaction profile. The exhaustive description being practically impossible, an idealized state description can only be obtained for the overall state. We mention however that this problem appears in an analogous way in physics, where in each model also only an idealized state description is obtained, depending on the possible experiments that are available. Hence, a specific property state corresponds, as in physics, to appropriate measurements that are available. In the case of the liar paradox entity, the exact meaning of the truth-state will be explained in the next section. The measurements that characterize the state of an entity referring to the classical-material layer are mostly straightforward. For example, analyzing if a piece of chalk is breakable has a fixed measurement-procedure namely, breaking the piece of chalk. This gives the corresponding entity a classical behavior in the sense that each time we perform the measurement we obtain, with certainty, the same result. In a quantum mechanical context we say that such entities are in an ‘eigenstate’ for the corresponding measurement. Also cognitive entities that do not refer to other layers can have straightforward measurement-procedures. Especially in the case of e.g. a mathematical theorem, there is often a straightforward procedure to establish a proof for it. And indeed each time we apply the same procedure, we obtain the same result, so we say again that those entities are in an eigenstate corresponding to this measurement procedure. Naturally, we do not expect everybody to know these mathematics, so the state of that same theorem will depend on what we call the ‘cognitive background’ of each cognitive person. Even mathematicians will have different opinions on the status of some theorems, therefore it is clear that the general cognitive background of people, over history, is important and plays a role in the nature of the states that will be attributed to cognitive entities. As this example points out, the nonfixed character of knowledge implies we accord states of nonfixed character to

cognitive entities. Again however, we remark that this is analogous to the physical situation. The state of a physical entity, defined correspondingly to measurement procedures, depends also on the general body of experimental possibilities at a certain epoch of time in the history of human culture, and hence is not fixed once and forever. Depending on whether this state has been defined by more ‘universal’ methods of experimentation, the state will approach in a deeper way the ontology of the entity. The same holds true for the state of a cognitive entity.

In the case of the liar paradox as a cognitive entity, for any measurement procedure, the truth-value will give us different results each time we intervene with it. Here, the state describing the truth-value will be called a superposition state as related to the measurement procedure in question, in analogy with the quantum mechanical concept. This means that we can not obtain a ‘certain’ prediction — in a classical sense — of properties, in this case ‘truth’ of ‘false’, of that cognitive entity. The input of the quantummechanical formalism is therefore appropriate.

How does one measure the liar paradox? The measurement here consists of two part-processes, ‘reading the sentence’ and ‘making a sentence true or false’. This means that in our description the liar paradox within the cognitive layer of reality is ‘in general’ — before the measurement — not in a state such that a reading would give true or false. The ‘true state’ and the ‘false state’ of the sentence are specific states; ‘eigenstates’ of the measurement. In general, the state of the liar paradox is not one of these two eigenstates. Due to the act of measurement, and in analogy with what happens during a quantum measurement, the state of the sentence changes (‘collapses’) into one of the two possible eigenstates, the ‘true state’ or the ‘false state’. This act of making a sentence true or false can be specifically described as ‘read it and make an hypothesis about its truth or falsehood’. In the next section we will apply this approach to the Double Liar, and see that an initial measurement followed by the sequence of logical inferences puts into work an oscillation dynamics that we can describe by a Schrödinger evolution over reasoning-time. We will also see in the next section, that the change of state due to measurement can be described by a projection operator in the quantum mechanical Hilbert space where the Schrödinger evolution is defined.

Once we make an hypothesis about one sentence the whole entity starts changing from one truth-state into another by continued reading with logical inference. When we stop this process, by means of not ‘looking at’ or ‘reasoning on’ the sentences any more, the entity — we hypothesise — reestablishes its original superposition state of indefiniteness. This super-

position state does not correspond to what has been called a truth value gap and does not assign a third truth value to the liar. In this sense, in our model, the set of semantical truth values, of which only one can be assigned to the cognitive entity, does not contain a third truth value or a value gap. Finally, we remark some interpretative issues concerning the origin of the entity's dynamics and the nature of the cognitive layer. From the quantummechanical analogy, the temporal evolution of the entity is expected to originate intrinsically, still the construction of the evolution — as will be clear from the next section — supposes the cognitive person's motivation by reasoning. We interpret the latter to be reflected in the autonomous dynamics, as such, the cognitive entity obtains its cognitive essence. The precise origin of temporal evolution has thereby become less transparent; the entity as well as the cognitive person will engender identical evolution. The nature of the cognitive layer, is essentially different from the sphere spanned by common material objects. In our approach — akin to 'effective' realism — the cognitive layer is 'Hilbert-space like', a personal and mental construct with social and cultural conditioning, a collective dynamic emergent layer carried by its cognitive participators. The extent and subtlety of this issue, allows in the present context merely explanatory simplifications. More detailed elaborations of the cognitive layer as an emergent, Hilbert-space like sphere in social groups are due (Aerts, Broekaert and Gabora, 1999).

3 The Liar-paradox: A Quantum Description of its Truth Behavior.

We will first discuss the Single Liar entity. By 'measuring' the single sentence we attribute a chosen truth-value to the sentence, immediately and logically inferring from its lecture the opposite truth-value. The cognitive person is therefore inclined to attribute in an alternating manner opposite truth values to the Single Liar sentence, until the 'measuring' process is chosen to be stopped. Subsequently no decisive and unambiguous truth-value can be attributed to the Single Liar sentence as it is. The application of the quantummechanical formalism suggests to describe this situation by a superposition of opposite truth-value states. We remark the striking correspondence between truth values and the two-fold eigenvalues of a spin-1/2 state of some quantum particles (e.g. an electron), and the oscillatory dynamics present in the reasoning dynamics respectively the evolution dynamics of a spin-1/2 particle in a constant magnetic field. This formal correspondence

will be used to construct a dynamical representation of the cognitive entity. Recall that this correspondence is possible, due to the fact that the spin of a particle is a quantized property, it exposes itself by means of distinct spin values. For a spin-1/2 particle there are only two distinct spin values namely ‘up’ and ‘down’ — appropriately corresponding to ‘true’ and ‘false’ values for the cognitive entity. The quantum mechanical formalism enables to express a superposition of these ‘up’ and ‘down’ states of a spin-1/2 particle by simple addition. When we apply the same idea to the single liar sentence, we obtain a state Ψ described by a pondered superposition of the two states of opposite truth-value:

$$\Psi = c_{true} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_{false} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The measurement itself, namely the interaction of the cognitive person on the entity when the sentence is being made true or being made false is described respectively by the true-projector P_{true} or false-projector P_{false} .

$$P_{true} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad P_{false} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

In this quantum mechanical description, the true-measurement (false measurement) on the superposed state Ψ results in the true state (resp. false state). The true state is represented as follows :

$$P_{true}\Psi = c_{true} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where the square modulus of the corresponding pondering factor c_{true} gives the statistical probability of finding the entity in the true-state. An unequivocal result is therefore not obtained when the superposition does not leave out one of the states completely, i.e. either c_{true} or c_{false} is zero. Only in those instances do we unambiguously attribute to a sentence its truth or falsehood.

We now look at the Double Liar sentences in more formal and mathematical detail. We consider three situations:

$$A \quad \begin{cases} (1) & \text{sentence (2) is false} \\ (2) & \text{sentence (1) is true} \end{cases}$$

$$B \quad \begin{cases} (1) & \text{sentence (2) is true} \\ (2) & \text{sentence (1) is true} \end{cases}$$

$$C \quad \begin{cases} (1) & \text{sentence (2) is false} \\ (2) & \text{sentence (1) is false} \end{cases}$$

From the case of the Single Liar we expect here a representation of the truth-behaviour by coupled \mathbb{C}^2 vectors, one for each sentence. Closer inspection of the coupled sentences of the two-sentence liar paradox of type (B) and (C), shows this is possible. The measurement of the Double Liar (B) allways will couple true-states of (B1) and (B2), and false-states of (B1) and (B2). In the case (C) on the other hand, measurement will couple the true-state of (B1) to the false-state of (B2), and the false-state of (B1) to the true-state of (B2). From a formal point of view, the equivalent spin-states in quantum mechanics would be described by the so called ‘singlet state’ and a ‘triplet state’ respectively. The singlet state indicates that two spin-1/2 particles are anti-alined and in an anti-symmetrical state, while a triplet state indicates that two spin-1/2 particles are alined and in a symmetrical state.

Whereas the Single Liar has been mathematically represented in a \mathbb{C}^2 finite dimensional complex Hilbert Space, we now need a $\mathbb{C}^2 \otimes \mathbb{C}^2$ space for the description of the (B) or (C) Double Liar. The tensorproduct \otimes connects the two sentences into one composed entity.

In the specific case of (C), and taking into account the anti-symmetric spin analog, Ψ is written as:

$$\frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

Still, in the application to the cognitive entity other choices of the pondering coefficients are possible. The only constraints on the coefficients are: equal amplitude and addition of the squared amplitudes to unity.

The state-vector for the liar paradox in case (B) can be constructed in a similar manner:

$$\frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

The projection operators which make sentence one and respectively sentence two true are now:

$$P_{1,true} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \mathbf{1}_2 \quad P_{2,true} = \mathbf{1}_1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

The projection operators that make the sentences false are obtained by switching the elements 1 and 0 on the diagonal of the matrix. These four projection operators represent the possible ‘logical’ interactions between the cognitive person and the cognitive entity.

During the continued measurement on the entities (B) and (C), the sequence of logical inferences results in a repetitive pattern of consecutive true-false states. These patterns are not very complicated, in case of entity (B) it will be a repetition of true-states (resp. false-states) depending on whether we presupposed an initial true (resp. false) state. While in the case of entity (C) it will always be an alternation between true-states and false states, no matter which state we presupposed.

Finally, we describe in detail the original double liar paradox, case (A). In this case we will show how the true-false cycle originates from the Schrödinger time-evolution of the appropriate initial state.

Instead of working within the coupled Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$, as in cases (B) and (C), we have to use a space of higher dimension for (A). This complexification is due to the fact that no initial state can be found in the restricted space $\mathbb{C}^2 \otimes \mathbb{C}^2$, such that application of the four true-false projection operators results in four orthogonal states respectively representing the four truth-falsehood states. The existence of such a superposition state — with equal amplitudes of its components — is required to describe the entity prior to, and after, any measurement procedure. If we perform the continued measurement on (A), by consecutive logical inference, the dynamical pattern is not anymore a two-step process like in the previous cases (B) and (C), instead we have a four-step process. Starting from the initial superposition state this four-step process can not be described by the coupled spin-1/2 models any more. In order to resolve this problem, recourse has to be taken to a 4 dimensional Hilbert-space for each sentence. The Hilbert-space needed to describe the Double Liar (A) is therefore $\mathbb{C}^4 \otimes \mathbb{C}^4$. The initial un-measured superposition state — Ψ_0 — of the Double Liar (A) is given by any equally pondered superposition of the four true-false states:

$$\frac{1}{2} \left\{ \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

Each next term in this superposition state is the consecutive state which is reached in the course of time, when the paradox is reasoned through. The truth-falsehood values attributed to these states, refer to the chosen measurement projectors.

Making a sentence true or false in the act of measurement, will be described by the appropriate projection operators in $\mathbb{C}^4 \otimes \mathbb{C}^4$. In the case we make sentence 1 (resp. sentence 2) true we get:

$$P_{1,true} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes \mathbf{1}_2 \quad P_{2,true} = \mathbf{1}_1 \otimes \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The projectors for the false-states are constructed by placing the 1 on the final diagonal place:

$$P_{1,false} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \otimes \mathbf{1}_2 \quad P_{2,false} = \mathbf{1}_1 \otimes \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As a consequence of making a freely chosen sentence of (A) either true or false, by logical inference the four consecutive states are repeatedly run through. In order to give a time-ordered description of this cyclic change of state, a continuous time t is introduced as an ordering parameter. The time-ordering parameter extrapolates the discrete moments of consecutive outcomes of the logical inferences, and as such relates to the physical time of reasoning. Under these interpretative restrictions a Schrödinger evolution over ‘time’ can be constructed.

Essentially, a Hamiltonian H can be constructed, such that the unitary evolution operator $U(t)$ — with $U(t) = e^{-iHt}$ — describes the cyclic change of logical inferences.

The construction of the evolution operator $U(t)$ is more easily accomplished by switching temporarily to an equivalent representation in a larger Hilbert space. Switching to a \mathbb{C}^{16} Hilbert-space is done without any modification or alteration of the problem, as it is isomorphic to the original $\mathbb{C}^4 \otimes \mathbb{C}^4$ coupled Hilbert space.

A new basis in \mathbb{C}^{16} is constructed from the basis of $\mathbb{C}^4 \otimes \mathbb{C}^4$ (i and j from 1 to 4) :

$$e_i \otimes e_j = e_{\kappa(i,j)} \quad \text{and} \quad \kappa(i,j) = 4(i-1) + j$$

Where κ is the natural basis transformation function from the $\mathbb{C}^4 \otimes \mathbb{C}^4$ to the \mathbb{C}^{16} Hilbert space, and the index from the new basis states in \mathbb{C}^{16} .

The initial superposition state Ψ_0 can now be represented in \mathbb{C}^{16} by:

$$\Psi_0 = \frac{1}{2} \{e_{10} + e_8 + e_{13} + e_3\}$$

For notational ease we continue to work further in a 4-dimensional subspace of \mathbb{C}^{16} , namely this subspace generated by the basis $(e_{10}, e_8, e_{13}, e_3)$. Obviously we do not loose any information by this restriction. The 4 by 4 submatrix — U_D — of the discrete unitary evolution operator, which describes the time-evolution at discrete instants of time when consecutive outcomes of logical inferences have been reached, is:

$$U_D = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

In order to obtain a description at every instance of the time-ordering parameter, a procedure of diagonalisation on the submatrix U_D is performed, i.e. $U_D|_{\text{diag}}$. The diagonalisation procedure allows to solve the matrix equation by breaking it into four uncoupled scalar equations. From the Schrödinger evolution and Stone's Theorem we obtain:

$$H_{sub}|_{\text{diag}} = i \ln U_D|_{\text{diag}}$$

Inverting the procedure of diagonalisation, the infinitesimal generator of the time-evolution — the submatrix hamiltonian — is obtained :

$$H_{sub} = \begin{pmatrix} -1/2 & -1/2 & (1-i)/2 & (1+i)/2 \\ -1/2 & -1/2 & (1+i)/2 & (1-i)/2 \\ (1+i)/2 & (1-i)/2 & 1/2 & 1/2 \\ (1-i)/2 & (1+i)/2 & 1/2 & 1/2 \end{pmatrix}$$

Or in terms of gamma matrices (these are merely introduced for shorthand notation for there is no implication of relativistic nature):

$$H_{sub} = \frac{1}{2} (-\gamma_0 - \gamma_5 + \gamma_0\gamma_1) + \frac{i}{2} (\gamma_1 + \gamma_2\gamma_3 + \gamma_0\gamma_5)$$

The submatrix of the evolution operator $U(t)$, valid at all intermediary times t is then given by the expression:

$$U_{sub}(t) = e^{-iH_{sub}t}$$

The time evolution operator $U_{sub}(t)$ in the 4-dimensional subspace of \mathbb{C}^{16} is:

$$U_{sub}(t) = \frac{1}{4} \left\{ \left(1 + e^{-it} + e^{it} + e^{2it} \right) 1 + i \left(1 - e^{-it} - e^{it} + e^{2it} \right) \gamma_2\gamma_3 \right. \\ \left. - \left(1 - e^{2it} \right) \gamma_5 + i \left(e^{-it} - e^{it} \right) \gamma_0\gamma_5 - i \left(-e^{-it} + e^{it} \right) \gamma_1 \right\}$$

In order to finalise our initial claim, we should bring back the hamiltonian H as well as the time-evolution operator $U(t)$ in the original $\mathbb{C}^4 \otimes \mathbb{C}^4$ Hilbert space. Although the outcome is straightforward to obtain by using the basis transformation function $\kappa(i, j)$, the complexity of the expression necessitates shorthand notation:

$$H = \sum_{\kappa, \lambda=1}^{16} H_{sub\kappa(i,j)\lambda(u,v)} O_{iu} \otimes O_{jv}$$

and

$$U(t) = \sum_{\kappa, \lambda=1}^{16} U_{sub\kappa(i,j)\lambda(u,v)}(t) O_{iu} \otimes O_{jv}$$

with;

$$O_{iu} \otimes O_{jv} = \{e_i \cdot e_u^t\} \otimes \{e_j \cdot e_v^t\}$$

For example, the term $\kappa = 3$, $\lambda = 10$ of the time evolution operator $U(t)$ is given by;

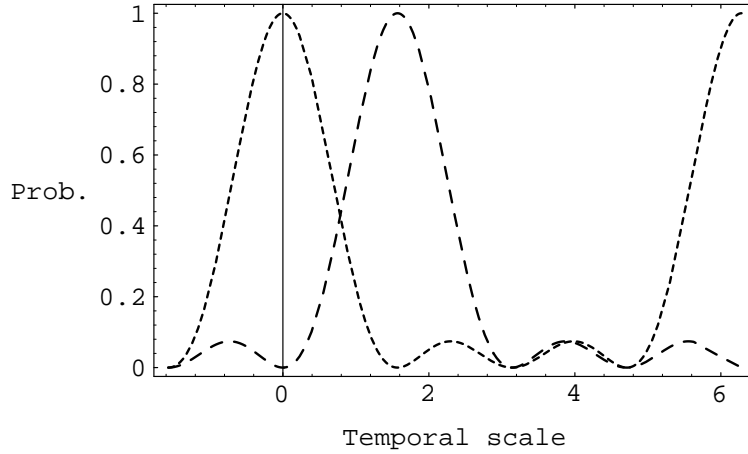
$$\frac{1}{4}(1 - ie^{-it} + ie^{-it} - ie^{2it}) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Finally, we complete the dynamical picture of the Double Liar cognitive entity (A); when submitted to any measurement at choice, the entity starts its truth-falsehood cycle, when left un-measured the entity remains statically in its undifferentiated superposition state. The latter statement follows immediately from the fact that the initial state Ψ_0 is left unchanged by the dynamical evolution $U(t)$;

$$\Psi_0(t) = \Psi_0$$

Ψ_0 is a time invariant, as it is an eigenstate of the Hamiltonian H . Exactly this time invariance points to the fact that the state Ψ_0 describes the existence of the cognitive entity (A) in its cognitive space, independent of any observer. The highly contextual nature of the Double Liar (A) — its unavoidable dynamics engendered by measuring it — implies, intrinsically it can not expose its complete nature, analogous to the quantum entities of the micro-physical world.

The evolution over the time-ordering parameter t of the truth behaviour of the cognitive entity during the measurement process can be illustrated graphically.



Temporal evolution of the cognitive evaluation of the Double Liar (A). At time-ordering parameter $t = 0$, a ‘false’-measurement on (A1) has been executed on the initial state Ψ_0 (short-dashed line). The second consecutive step in the logical inference, the ‘true’-state of (A2) at time-ordering parameter $t = \frac{\pi}{2}$, and the intermediary times, is shown by the long-dashed line. For clarity, consecutive steps have been deleted. Discrete moments of outcomes of logical inferences are at time-ordering parameter $t = n\frac{\pi}{2}$

4 Conclusion.

We found that the quantum mechanical formalism can be applied to self-referent cognitive entities as the single liar and double liar paradox. Essentially the two necessary features of the dynamics were constructed; when measured the entity starts its truth-falsehood cycle, when left un-measured the entity remains invariantly in its initial state. A measurement of the entity — engendered by appropriating one sentence its truth or falsehood — sets into action the dynamical evolution which attributes, alternatively over time, truth and falsehood to the coupled sentences. The unmeasured entity Ψ_0 does not change over time, its invariance reflects its emergent nature.

The truth-falsehood alternation, due to a measurement process, and the time-invariance of the initial state are both derived from constructed evolution operator $U(t)$, driven by the Hamiltonian H . The quantum formalism therefore has proven an appropriate tool to describe the liar paradox entity.

We set out with the idea of realist cognitive entity, if it can be generally and

cognitively influenced on as a practically stable configuration, and by the limited number of different states that it can be in. We put forward that exactly this time invariance points to the fact that the state Ψ_0 describes the existence of the cognitive entity in its cognitive space, independent of any observer. For this reason we take the position at liberty to put forward a realistic picture of the cognitive reality, in the sense that $\Psi_0(t)$ represents the state of the real entity.

The nature of the cognitive layer, as here proposed, is essentially different from the space of common macroscopic material objects. The cognitive layer is ‘Hilbert-space like’, and originates from the cognitive person. Quite different from the complexity-theory approach, which emphasizes dynamics, the present formalism clearly describes the emergent entity as an ontological state. An important question in our research therefore remains; the relation between the quantum mechanical state description and the non-linear approach of complex dynamics. Rather speculatively some relation between privileged states in both models could be expected. A model exposing common grounds to both formalisms could in first approach allow inquiry into the relation between the quantum mechanical eigenstates of the Hamiltonian and the phase-space attractors of the corresponding non-linear evolution equations. Eventually, more detailed elaborations of the cognitive layer as an emergent, Hilbert-space like sphere in social groups, can relate both approaches.

The generalisation to other cognitive entities with reference to non-cognitive entities or processes can most probably, to extent, be covered by the ϵ -model formalism (Aerts, 1986, Aerts and Durt 1994, Aerts, Durt and Van Bogaert 1993). This model allows to describe effects of intermediary contextuality, which is expected in common cognitive entities. Further development needs to bring into the formalism, a general dynamic, and contextual influences other than those of the cognitive person on the truth behavior of the cognitive entities.

Philosophical questions, quite speculative at this stage of our research, can be put forward: e.g. Can we, from the example of the Liar paradox, learn something in general about the nature and origin of dynamical change? What real life experiments can underpin the realistic approach to the cognitive entities?

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